

Isomorphic Properties among the Connectivity of Various Graphs

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Abstract

The various types of identical graphs having molecular structure of different degrees are isomorphic. In this paper, we study the structure of two graphs, which are isomorphic. Among the class of trees (star like tree) unicyclic, bicyclic, tricyclic, tetracyclic, pentacyclic, hexa-cyclic, heptacyclic etc. are isomorphic. Under discussion two graphs G_1 and G_2 are isomorphic because both graphs have same number of vertices and edges. We also point out different important applications of the various types of isomorphic graphs. In whole, we study about two graphs that can exist in different forms having the same number of vertices, edges having same degrees and the same edge connectivity. Such graphs called isomorphic graphs. While graph plotting and graph representation are sufficient topic in graph theory. In order to concentrate only on the abstract structure of graphs. A graph property defined to be a property to keep safe under all isomorphic of a graph. By brief look, we see that two graphs may appear very different when inspected visually, but they could have the same adjacency structure.

Keywords: Isomorphic, graphs, vertex, edge.

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1. Introduction

The theory of graphs has a vast and rich history [1]. In speculative chemistry and biology, molecular framework [2] have used for quantitative information about molecule [3]. The fundamental axiom of isomorphic is a heuristic assumption, which explain the relation of nature between the remarkable experience and brain processes. It first suggested by Wolfgang Kohler (1920). His deepest commitments were theoretical and philosophical but he has made many important contributions in field of science. By utilized scientific methods to study the following earlier formulations by G.E Muller (1896) and Max. Wertheimer (1912). Sometimes the graph isomorphism becomes hard because these related topic problems are computational [4]. In order to determine whether two finite graphs are a property P is called an isomorphic invariant if given any graph G_1 and G_2 . If G_1 has property P and G_1 is isomorphic to G_2 then G_2 has property P [5]. Often two graphs are not isomorphic because their graph never change number of vertices, number of edges, and degree of vertices all match.

We can check two given graphs are isomorphic if and only if both have equal number of vertices, equal number of edges, degree of sequence is same and number of circuits of specific length are same [6]. Two graphs said to be isomorphic if both are perhaps the same graphs. Just drawn differently with different name [7]. Isomorphic graphs have identical behavior for any graph theoretic properties formally speaking [8]. Both two graphs $G_1(V, E)$ and $G_2(V, E)$ are

identical if $V_1 = V_2$ and $E_1 = E_2$. In other words, we can say that both graphs are said to be isomorphic if they have same structure. Both graphs G_1 and G_2 said to be isomorphic if there is one-to-one correspondence between their vertices and edges incident relationship is specific. More commonly two graphs G_1 and G_2 are isomorphic because both graphs have the same number of vertices and edges. Each distinguishable vertex in G_2 corresponding to each vertex in G_1 then in G_2 there must be an edge in G_2 . In whole we write it as $a \rightarrow u$, $b \rightarrow v$, $c \rightarrow x$, $d \rightarrow w$ vertex corresponding to u and vertex corresponding to v. Similarly, $|ab| = |uv|$, $|db| = |wv|$, $|cd| = |xw|$, $|ac| = |ux|$, $|bc| = |vx|$ edges corresponding to u and edges corresponding to v so $G_1 \approx G_2$ or $G_1(V_1, E_1) \approx G_2(V_2, E_2)$.

2. Some Important and Basic Idea about Isomorphism

Graphs are generally used to mix constitutional data in many fields, in conjunction with computer fertility and arrangements awareness, and graph pairing i.e., description similarities between graphs. Bra is an important tool in these areas, as the graph isomorphism problem is known as the exact graph matching. Not only are the graphs being isomorphic but also both graphs are really the same graphs. [9]. It surely helps to work out the actual definitions of isomorphic graph. In the completely two graphs are isomorphic if and only if their complement graphs are isomorphic [10]. In case of matrices, two graphs are isomorphic if their adjacent matrices are same. Two graphs

are isomorphic if there is a graph obtained by rubbing out some vertices of one graph and therefore only in the other graph for isomorphic [11].

3. Results and discussion

Two graphs are isomorphic if and only if there complement graphs are isomorphic. Two graphs are isomorphic if there are just and see matrices are the same if deleting some vertices of one graph and their corresponding meat in their graphs are isomorphic by obtaining corresponding sub graphs. Therefore, graph isomorphism is a phenomenon of existing the same graph in more than one form. Examples: some graphs existing in multiple forms called isomorphic graphs. A pair of graphs are set to be isomorphic if both have the same structures. Both under discussion graphs have some graphs just drawn differently with the different names. They have a denticle behavior for any graphical properties [12]. Both cross satisfy injective property as well as surjective property. If the above-mentioned two properties are satisfied then it called by bijective function [13-14]. As in both graph two vertices V_1^{\square} and V_2^{\square} are adjacent if, $G_1^{\square}, G_1^{\square} \in E(G)$ if and only if $\phi_1^{\square}(V_1^{\square})\phi_2^{\square}(V_2^{\square}) \in E(H)$. If These two graphs G and H are isomorphic if there exist a bijection:

$$\psi; V(G) \rightarrow V(H) \text{ such that } uv \in E(G) \text{ if and only if } \psi(u), \psi(v) \in E(H).$$

In this case, we can write $G \cong H$,

$$uv \in E(H) \rightarrow \psi(u)\psi(v) \in E(G) \\ uv \notin E(G) \rightarrow \psi(u), \psi(v) \notin E(H)$$

In two cases both graphs are not, isomorphic we can write $G \not\cong H$. the order of the graph is the number of vertices in the graphs. In above graph $O(G) = 6$. The size of the graph is the number of edges in the graph the above graph size is 6. Tree example problem on isomorphism in discrete mathematics.

Example 1: Check whether there are two graphs of tree are

1. Number of vertices of $G_1^{\square} = \text{No. of vertices of } G_2^{\square} = 5$
2. Number of edges of $G_1^{\square} = \text{number of edges of } G_2^{\square} = 4$
3. Degree sequence of $G_1^{\square} = \text{Degree sequence of } G_2^{\square} = (1, 2, 3, 1, 1) = (1, 1, 1, 2, 3)$
4. One to one mapping of $G_1^{\square} = \text{of one-to-one mapping of } G_2^{\square} = (1, 1, 2, 3)$.

Hence one to one correspondence between G_1^{\square} and G_2^{\square}

5. Edge preserving property of $G_1^{\square} = \text{Edge preserving property of } G_2^{\square}$

$$P - Q \Leftrightarrow D - E \quad A - B \Leftrightarrow T - R \\ P - R \Leftrightarrow D - B \quad B - C \Leftrightarrow R - S \\ R - S \Leftrightarrow B - C \quad B - D \Leftrightarrow R - P \\ S - T \Leftrightarrow E - A \quad D - E \Leftrightarrow P - Q$$

6. Adjacent matrix of G_1^{\square}

Adjacent of matrix G_2^{\square}

Example 2: Check whether these two graphs are isomorphic or not?

1. Number of vertices of $G_1^{\square} = \text{number of vertices } G_2^{\square} = 8$
2. Number of edges of $G_1^{\square} = \text{number of edges of } G_2^{\square} = 7$
- 3.

$G_1^{\square} (V_1^{\square}, E_1^{\square})$ adjacency list \neq

$G_2^{\square} (V_2^{\square}, E_2^{\square})$ adjacency list

$$V_1^{\square} : V_3^{\square} \quad (\text{Deg1}) \\ V_1^{\square} : V_3^{\square} \quad (\text{Deg1})$$

$$V_2^{\square} : V_3^{\square} \quad (\text{Deg1})$$

$$V_2^{\square} : V_3^{\square} \quad (\text{Deg1})$$

$$V_3^{\square} : V_1^{\square}, V_2^{\square} : V_4^{\square} \quad (\text{Deg3})$$

$$V_3^{\square} : V_1^{\square}, V_2^{\square} : V_4^{\square} \quad (\text{Deg3})$$

$$V_4^{\square} : V_3^{\square}, V_5^{\square} \quad (\text{Deg2})$$

$$V_4^{\square} : V_3^{\square}, V_5^{\square} : V_6^{\square} \quad (\text{Deg3})$$

$$V_5^{\square} : V_4^{\square}, V_6^{\square} : V_7^{\square}, V_8^{\square} \quad (\text{Deg4})$$

$$V_5^{\square} : V_4^{\square} \quad (\text{Deg1})$$

$$V_6^{\square} : V_5^{\square} \quad (\text{Deg1})$$

$$V_6^{\square} : V_4^{\square}, V_7^{\square} \quad (\text{Deg2})$$

$$V_7^{\square} : V_5^{\square} \quad (\text{Deg1})$$

$$V_7^{\square} : V_6^{\square}, V_8^{\square} \quad (\text{Deg2})$$

$$V_8^{\square} : V_5^{\square} \quad (\text{Deg1})$$

$$V_8^{\square} : V_7^{\square} \quad (\text{Deg1})$$

Degree sequence of $G_1^{\square}(V_1^{\square}, E_1^{\square})$ and

$G_2^{\square}(V_2^{\square}, E_2^{\square})$ are different so both are not isomorphic.

Specific definition of isomorphic graph.

Definition: The graphs $G_1^{\square} = (V_1^{\square}, E_1^{\square})$ and $G_2^{\square} = (V_2^{\square}, E_2^{\square})$

are called isomorphic if there is one to one function from

V_1^{\square} to V_2^{\square} with the property that “u” and “v” are adjacent in

G_1^{\square} , for all “u” and “v” in. Suppose that edge ‘e’ is incident

on V_1^{\square} and V_2^{\square} in G then the corresponding edge in “G” must

be incident on vertices “ V_1^{\square} ” and “ V_2^{\square} ” that corresponds

to $V_1^{\square}, V_2^{\square}$

Example 3: Check whether a unicyclic graph G

corresponding to a graph H is isomorphic or not. Mapping

from f: $v \rightarrow u$ such that:

$$f(V_1^{\square}) = U_1^{\square}$$

$$f(V_3^{\square}) = U_3^{\square}$$

$$f(V_2^{\square}) = U_4^{\square}$$

$$f(V_4^{\square}) = U_2^{\square}$$

Which is one to one function mapping

$$(V_1^{\square}, V_2^{\square}) \cong (U_1^{\square}, U_2^{\square})$$

$$(V_1^{\square}, V_3^{\square}) \cong (U_1^{\square}, U_3^{\square})$$

$$(V_3^{\square}, V_2^{\square}) \cong (U_3^{\square}, U_4^{\square})$$

$$(V_2^{\square}, V_4^{\square}) \cong (U_4^{\square}, U_2^{\square})$$

All the vertices of both graph G and H are adjacent.

Hence, both graph G and H are isomorphic by holding these properties.

1. Number of vertices of $G_1^{\square} = \text{number of vertices of } G_2^{\square} = 4$
2. Number of edges of $G_1^{\square} = \text{number of edges of } G_2^{\square} = 4$
3. Degree sequence of vertices of $G_1^{\square} = \text{Degree sequence of vertices of } H$

$$d(V_1^{\square}) = d(V_2^{\square}) = d(V_3^{\square}) = d(V_4^{\square}) = 2$$

$$d(U_1^{\square}) = d(U_2^{\square}) = d(U_3^{\square}) = d(U_4^{\square}) = 2$$

4. Edge preserving property of $G = \text{Edge preserving property of } H$

$$V_1^{\square} - V_2^{\square} \Leftrightarrow U_1^{\square} - U_4^{\square}$$

$$U_1^{\square} - U_4^{\square} \Leftrightarrow V_1^{\square} - V_2^{\square}$$

$$V_2^{\square} - V_4^{\square} \Leftrightarrow U_4^{\square} - U_2^{\square}$$

$$U_4^{\square} - U_2^{\square} \Leftrightarrow V_2^{\square} - V_4^{\square}$$

$$V_4^{\square} - V_3^{\square} \Leftrightarrow U_2^{\square} - U_3^{\square}$$

$$U_2^{\square} - U_3^{\square} \Leftrightarrow V_4^{\square} - V_3^{\square}$$

$$V_3^{\square} - V_1^{\square} \Leftrightarrow U_3^{\square} - U_1^{\square}$$

$$U_3^{\square} - U_1^{\square} \Leftrightarrow V_3^{\square} - V_1^{\square}$$

So, all the above-mentioned properties in the graph held so the given two graphs G and H are isomorphic.

Example 4: Check whether two linear independent cyclic graphs of degree 4 are isomorphic.

- No. of vertices of $G_1^{\square} \square_{\square}^{\square} =$ No. of vertices of $G_2^{\square} \square_{\square}^{\square} = 5$
- No. of edges of $G_1^{\square} \square_{\square}^{\square} =$ No. of edges of $G_2^{\square} \square_{\square}^{\square} = 5$
- $G_1^{\square} (V_1^{\square}, E_1^{\square})$ adjacency list =
 $G_2^{\square} (V_2^{\square}, E_2^{\square})$ adjacency list
 $d(a)=1$ $d(t)=1$
 $d(b)=2$ $d(q)=2$
 $d(c)=2$ $d(p)=2$
 $d(d)=2$ $d(s)=2$
 $d(e)=3$ $d(r)=3$
- Edge preserving property of $G_1^{\square} =$ Edge preserving property of G_2^{\square}

- $a - e \Leftrightarrow t - r$ $t - r \Leftrightarrow a - e$
 $e - d \Leftrightarrow r - s$ $r - s \Leftrightarrow e - d$
 $e - p \Leftrightarrow p - r$ $p - r \Leftrightarrow c - e$
 $b - d \Leftrightarrow s - q$ $s - q \Leftrightarrow b - d$
 $b - c \Leftrightarrow p - q$ $p - q \Leftrightarrow b - c$

5. Hence the adjacent matrix of G_1^{\square} Adjacent matrix of G_2^{\square}

Example 5: Check whether two identical tricyclic graphs are isomorphic Identical Graph.

Two graphs $G_1^{\square} = (V_1^{\square}, E_2^{\square})$ and $G_2^{\square} = (V_2^{\square}, E_2^{\square})$ are identical if $V_1^{\square} = V_2^{\square}$ and $E_1^{\square} = E_2^{\square}$

3.1. Isomorphic Graph

Two graphs said to be isomorphic if they have same structure. These two graphs G_1^{\square} and G_2^{\square} are isomorphic because both graphs have same number of vertices and edges. Each distinct vertex in G_2^{\square} corresponding to each vertex in G_1^{\square} .

Example 6: Verify the whether the following two graphs are isomorphic or not?

Solution After labeling vertices randomly, the above two graph G and H have the following properties

- No. of vertices of $G_{\square}^{\square} \square_{\square}^{\square} =$ No. of vertices of $H_{\square}^{\square} \square_{\square}^{\square} = 5$
- No. of edges of $G_{\square}^{\square} \square_{\square}^{\square} =$ No. of vertices of $H_{\square}^{\square} \square_{\square}^{\square} = 6$
- Degree sequence of G = Degree sequence of H = 2, 3, 2, 2, 3
- Circuit of length 3 in G and H
- Circuit of length 4 in G and H
- Circuit of length 5 in G and H.

In G, e-b-c- ; In H, u-y-v-u
 In G, e-d-a-b-e; in H, u-y-v-u
 In G, a-b-c-e-d-e, in H, y-x-w-v-y the two graphs have same adjacency matrices re-label the vertices which justification of rules played by vertices. There are two vertices of degree 3 in G and H. b and e in G and y, v in H. There are 3 vertices of degree 2 in G and H. u, x, w in H and a, d, c in G. Both play the same role in both graphs and each side have degree 3.

So, $u \leftrightarrow c$ u must be mapped with c
 $e \leftrightarrow y, b$

$\leftrightarrow v$ these two vertices are mapped with b and x, v and d in w.

$a \leftrightarrow w$ and $d \leftrightarrow x$.

Therefore, all the invariant properties of isomorphic graphs are satisfied. If we Label the vertices a b c d e in G as $u_1^{\square}, u_2^{\square}, u_3^{\square}, u_4^{\square}, u_5^{\square}$ and H we have to label the vertices as follows w, v, u, x, y in H as $v_1^{\square}, v_2^{\square}, v_3^{\square}, v_4^{\square}, v_5^{\square}$.

Example 7: Verify the whether the following two graphs are isomorphic or not

Solution

Verify property invariant properties invariant properties

- The number of vertices in G = Number of vertices in H = 6
- The number of edges in G = the number of edges in H = 7
- The number of degree sequence in G = 2, 3, 2, 3, 2, 2
The number of degree sequence in H = 2, 2, 3, 2, 3, 2
2 degrees and 4 vertices in G 2 degrees and 4 vertices in H
- Circuit of length 3: Nil in both graphs
- Circuit of length 4

$$u_1^{\square} - u_2^{\square} - u_3^{\square} - u_4^{\square} - u_1^{\square} \text{ in G}$$

$$v_3^{\square} - v_4^{\square} - v_5^{\square} - v_6^{\square} - v_3^{\square} \text{ in H}$$

Circuit of length 5

$$u_2^{\square} - u_3^{\square} - u_4^{\square} - u_5^{\square} - u_6^{\square} - u_2^{\square} \text{ in G}$$

$$u_1^{\square} - u_2^{\square} - u_3^{\square} - u_4^{\square} - u_1^{\square} \text{ in G}$$

Two circuit in G

$$v_1^{\square} - v_2^{\square} - v_3^{\square} - v_6^{\square} - v_5^{\square} - v_1^{\square} \text{ in H}$$

$$v_1^{\square} - v_2^{\square} - v_3^{\square} - v_4^{\square} - v_5^{\square} - v_1^{\square} \text{ in H}$$

Two circuit in H

u_6^{\square} adjacent with u_2^{\square} and u_5^{\square} in G

u_6^{\square} Adjacent with u_5^{\square} and u_2^{\square} in G

$u_2^{\square} \rightarrow v_3^{\square}$ and $u_4^{\square} \rightarrow v_5^{\square}$

v_6^{\square} Also adjacent with v_3^{\square} and v_5^{\square} in H

v_6^{\square} adjacent with v_3^{\square} and v_5^{\square} in H

$u_1^{\square} \rightarrow v_6^{\square}$ and $u_3^{\square} \rightarrow v_4^{\square}$

Adjacent with degree 3 vertex u_4^{\square} one side and degree 2 vertex u_6^{\square} other side which is adjacent with v_5^{\square} on one side and a degree 2 vertex other side in H.

v_1^{\square} is such a vertex so

$$u_5^{\square} \rightarrow v_1^{\square}$$

Finally, no other option

$$u_6^{\square} \rightarrow v_2^{\square}$$

Relabel the vertices

$$u_1^{\square}, u_2^{\square}, u_3^{\square}, u_4^{\square}, u_5^{\square}, u_6^{\square} \text{ as } a, b, c, d, e, f \text{ in G}$$

$$\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$$

$$v_6^{\square}, v_3^{\square}, v_4^{\square}, v_5^{\square}, v_1^{\square}, v_2^{\square} \text{ as } u, v, w, x, y, z \text{ in H.}$$

Since $A_G^{\square} \equiv A_H^{\square}$ so G and H are isomorphism between G and H established. Incident matrix in undirected graph.

Let G be a graph with n vertices and no self-loop. Define n-by-c matrix $A = [a]_{ij}$ whose n rows corresponds to the end vertices and the e column to the edge as follows

$[a]_{ij}^{\square} = 1$ if jth edge is incident ith vertex v_i^{\square}

Incident = 0 otherwise

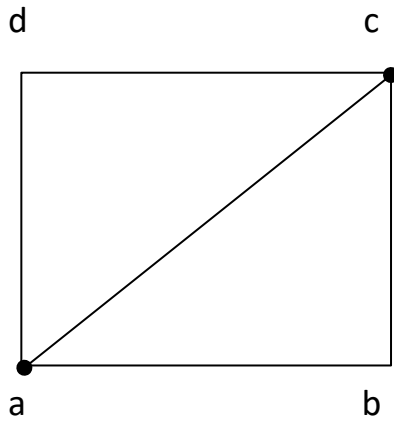
Example 8: Under consideration matrix is incident matrix and called binary matrix or (0, 1) matrix and also called vertex edge incidence matrix and complete matrix.

Degree of vertices shown below.

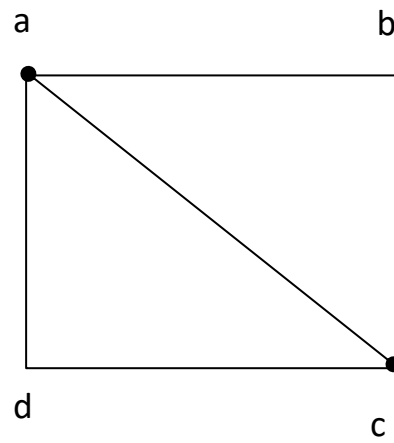
$Deg(A) = 3, Deg(B) = 2, Deg(C) = 2, Deg(D) = 3.$

In case of column, we have:

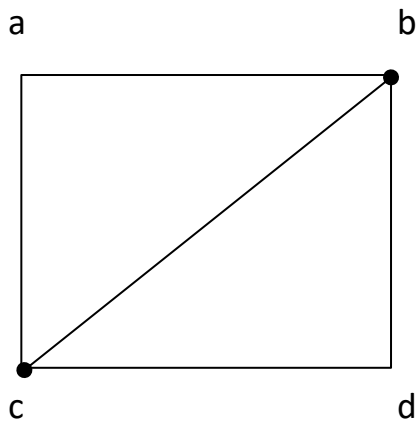
Every column has two "1" exactly and its sum always 2 i.e., $1+1=2$ If reduced D row, then we achieve a matrix called a reduced incidence matrix. Conversely if we have to obtained a reduced to incidence matrix then we have to add 1 or 0 to obtained D row then obtain matrix is called reduced incidence matrix.



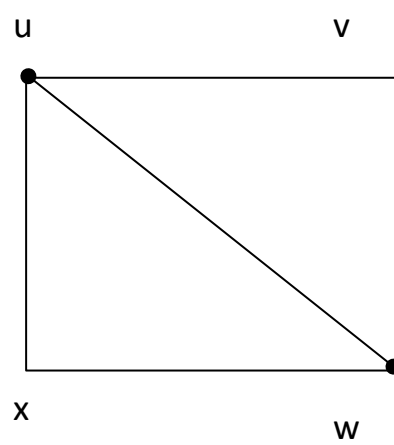
$G_1(V_1, E_1)$



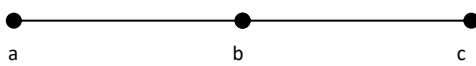
$G_2(V_2, E_2)$



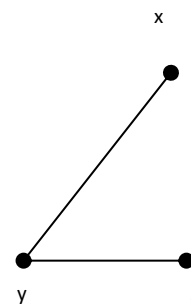
$G_1(V_1, E_1)$



$G_2(V_2, E_2)$



G

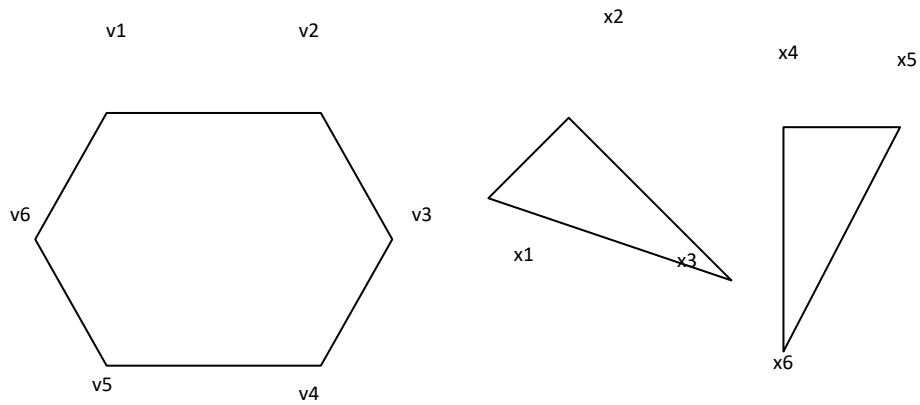


H

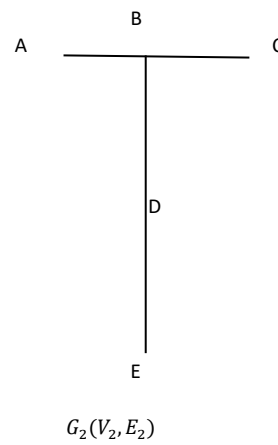
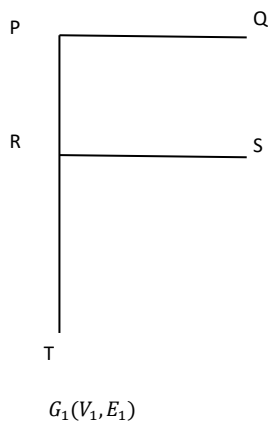
As $a \rightarrow x$, $b \rightarrow y$, $c \rightarrow z$ then we can say $G \equiv H$ or $H \equiv G$

$$\phi: V(G) \rightarrow V(H)$$

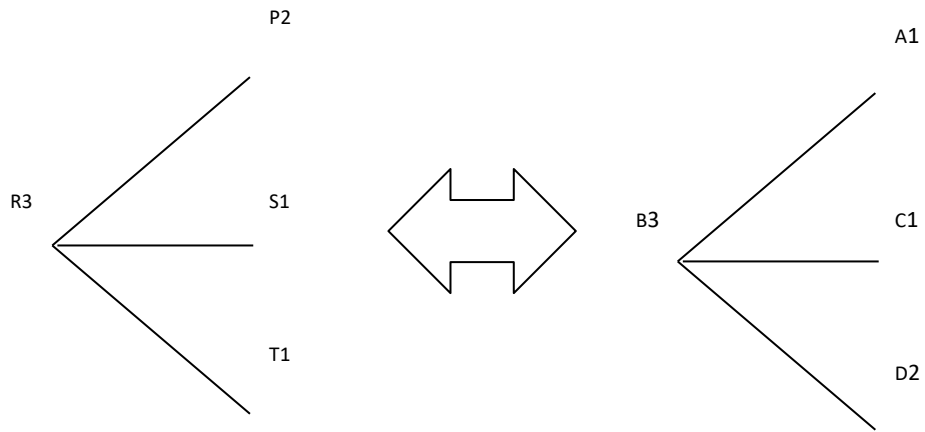
$$\phi: \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix}$$



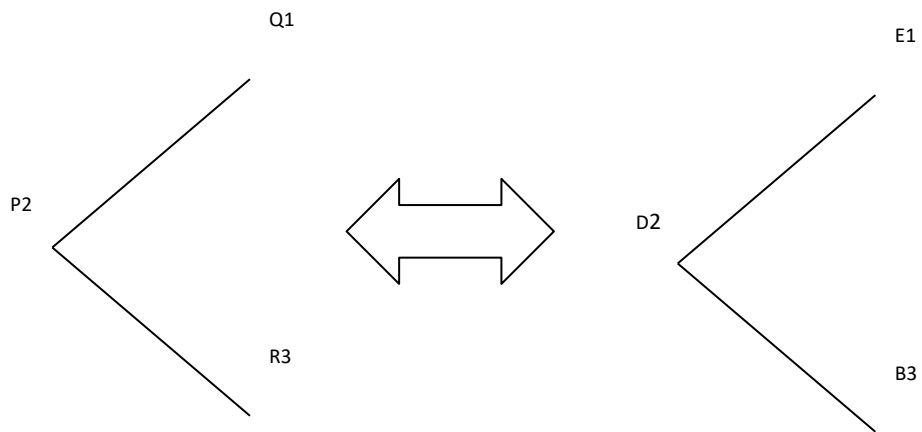
$$\phi: \begin{bmatrix} V_i & V_j & V_k \dots \\ X_1 & X_2 & X_3 \dots \end{bmatrix}$$



Example 1. Check whether there are two graphs of tree are isomorphic or not?



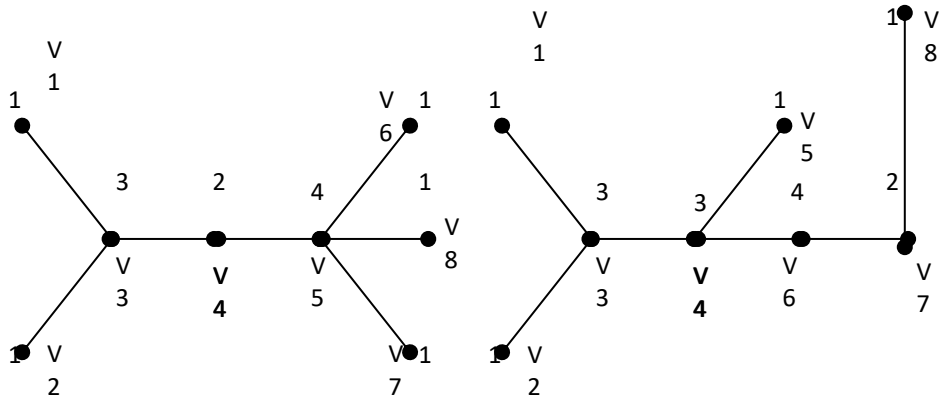
$$R \Leftrightarrow B, Q \Leftrightarrow E, R \Leftrightarrow B$$



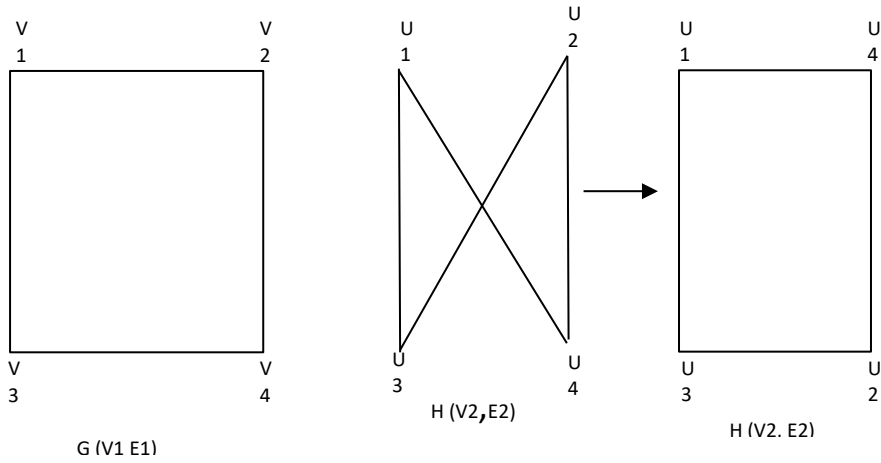
One to one correspondence between G_1^{\square} and G_2^{\square}

$$\begin{array}{c}
 \begin{array}{ccccc}
 & P & Q & R & S & T \\
 P & 0 & 1 & 1 & 0 & 0 \\
 Q & 1 & 0 & 0 & 0 & 0 \\
 R & 1 & 0 & 0 & 1 & 1 \\
 S & 0 & 0 & 1 & 0 & 0 \\
 T & 0 & 0 & 1 & 0 & 0
 \end{array}
 &
 \begin{array}{ccccc}
 & D & E & B & C & A \\
 D & 0 & 1 & 1 & 0 & 0 \\
 E & 1 & 0 & 0 & 0 & 0 \\
 B & 1 & 0 & 0 & 1 & 1 \\
 C & 0 & 0 & 1 & 0 & 0 \\
 A & 0 & 0 & 1 & 0 & 0
 \end{array}
 \end{array}$$

Example 1. G_1^{\square} and G_2^{\square} are isomorphic graphs.



Example 2. Check whether these two graphs are isomorphic or not?

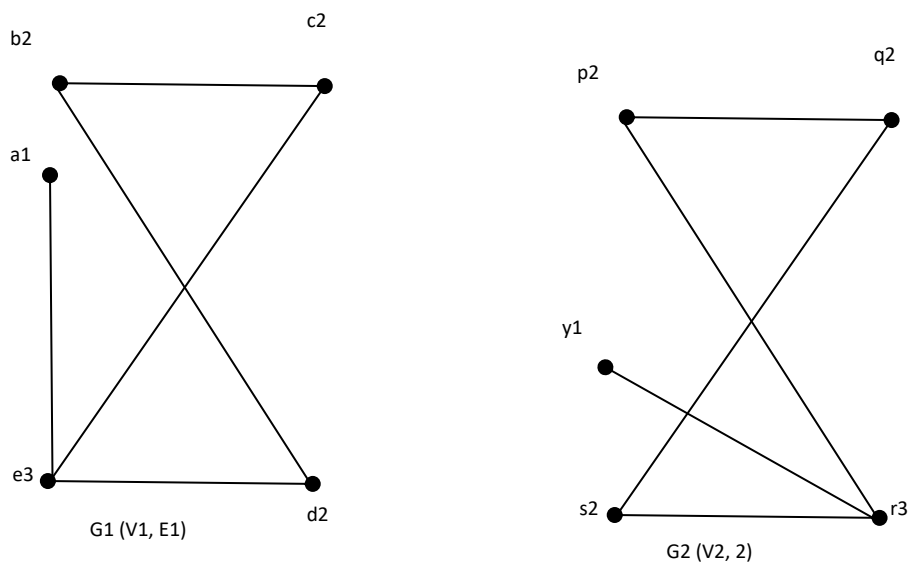


Example 2. The graphs $G_1^{\square} = (V_1^{\square}, E_1^{\square})$ and $G_2^{\square} = (V_2^{\square}, E_2^{\square})$ are called isomorphic if there is one to one function from V_1^{\square} to V_2^{\square} with the property that “u” and “v” are adjacent in G_1^{\square} , for all “u” and “v” in.

$$\begin{matrix}
 & V_1^{\square} & V_2^{\square} & V_3^{\square} & V_4^{\square} \\
 \begin{matrix} V_1^{\square} \\ V_2^{\square} \\ V_3^{\square} \\ V_4^{\square} \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}
 \end{matrix}$$

$$\begin{matrix}
 & U_1^{\square} & U_4^{\square} & U_3^{\square} & U_2^{\square} \\
 \begin{matrix} U_1^{\square} \\ U_4^{\square} \\ U_3^{\square} \\ U_2^{\square} \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}
 \end{matrix}$$

Example 3. Edge preserving property of G=Edge preserving property of H

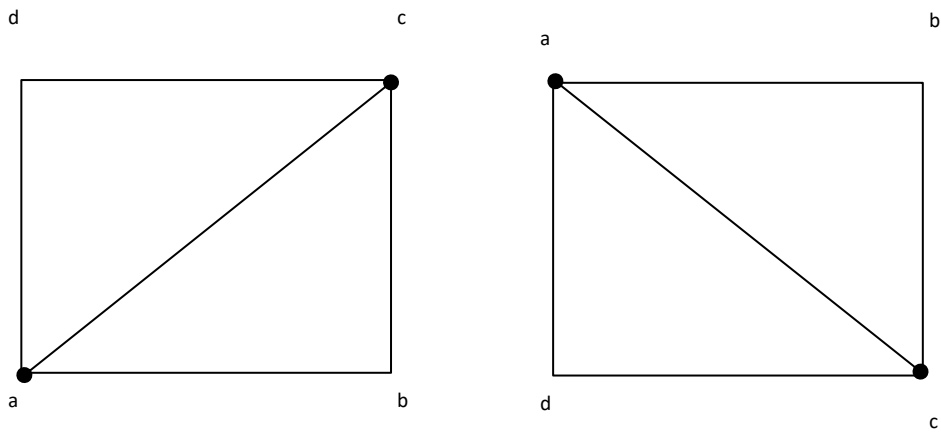


Example 4. Check whether two linear independent cyclic graphs of degree 4 are isomorphic.

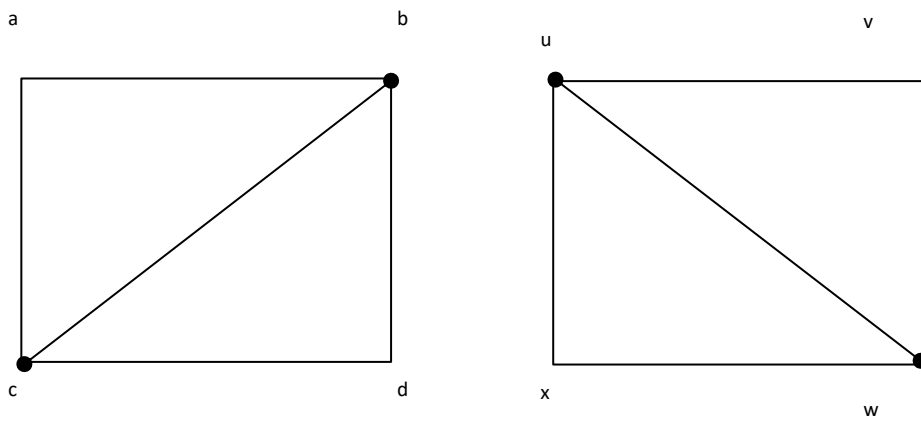
$$\begin{array}{c}
 \begin{array}{ccccc}
 & a & b & c & d & e \\
 a & \left[\begin{array}{ccccc}
 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & 1 & 1 & 0 \\
 0 & 1 & 0 & 0 & 1 \\
 0 & 1 & 0 & 0 & 1 \\
 1 & 0 & 1 & 1 & 0
 \end{array} \right]
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{ccccc}
 & t & q & p & s & r \\
 t & \left[\begin{array}{ccccc}
 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & 1 & 1 & 0 \\
 0 & 1 & 0 & 0 & 1 \\
 0 & 1 & 0 & 0 & 1 \\
 1 & 0 & 1 & 1 & 0
 \end{array} \right]
 \end{array}
 \end{array}$$

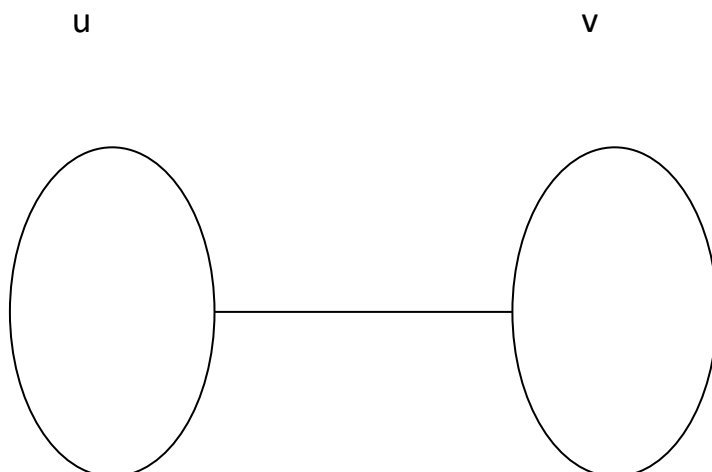
Example 4. G_1^{\square} and G_2^{\square} are isomorphic graphs



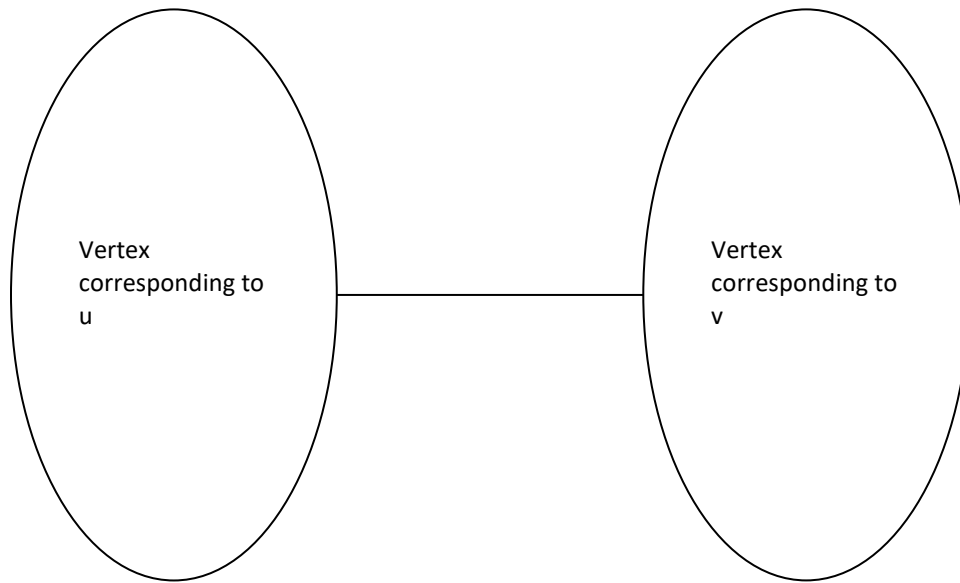
Example 5. Two graphs $G_1^{\square} = (V_1^{\square}, E_2^{\square})$ and $G_2^{\square} = (V_2^{\square}, E_2^{\square})$ are identical if $V_1^{\square} = V_2^{\square}$ and $E_1^{\square} = E_2^{\square}$



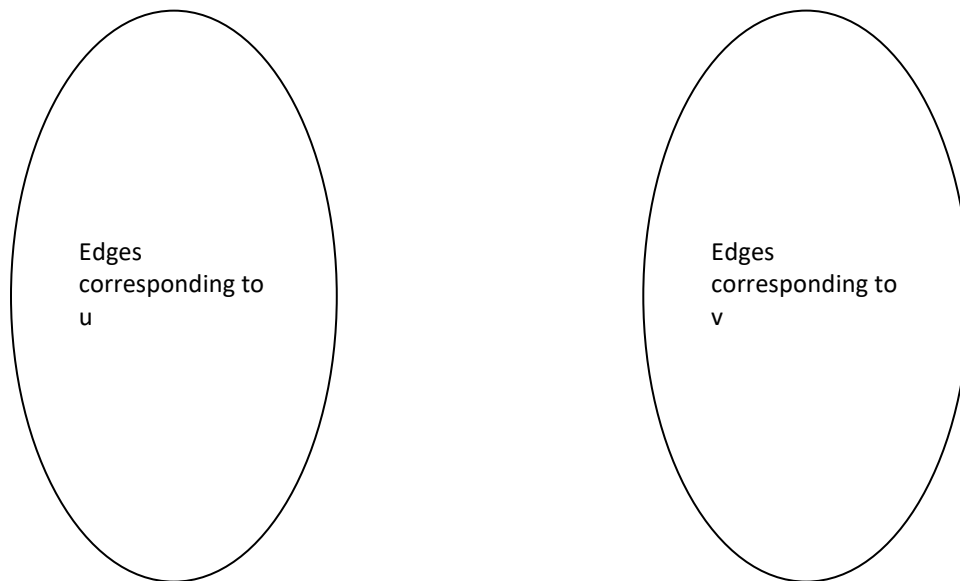
These two graphs G_1^{\square} and G_2^{\square} are isomorphic because both graphs have same number of vertices and edges.



Then in G_2^{\square} there must be an edge in G_2^{\square}



$$a \rightarrow u, b \rightarrow v, c \rightarrow x, d \rightarrow w$$

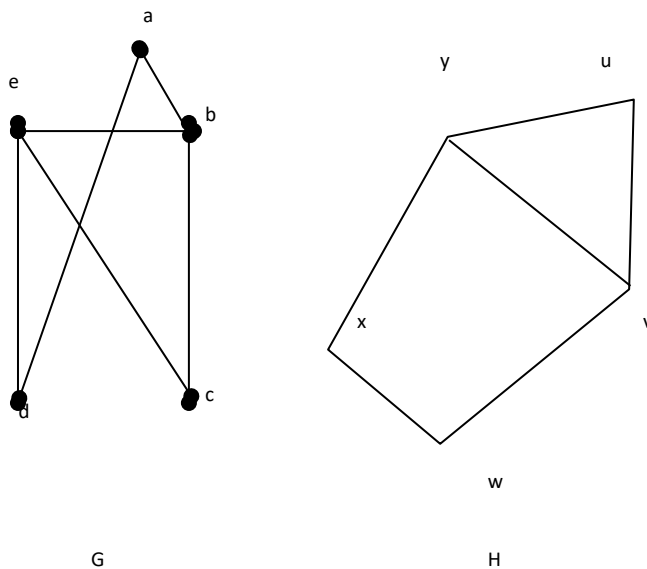


Edge corresponding u

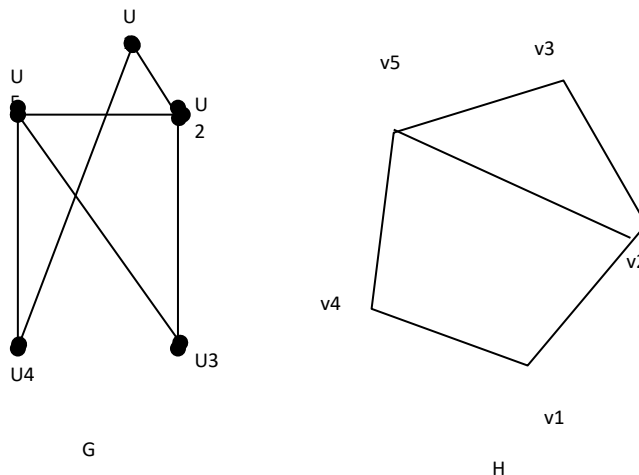
Edge corresponding to v

$$|ab|=|uv|, \quad |db|=|wv|, \quad |cd|=|xw|, \quad |ac|=|ux|, \quad |bc|=|vx|$$

Therefore, 2 graphs $G_1^{\square} = G_2^{\square}$ or graphs $G_1^{\square} = (V, E) = G_2^{\square} = (V, E)$.



Example 6. Verify the whether the following two graphs are isomorphic or not?



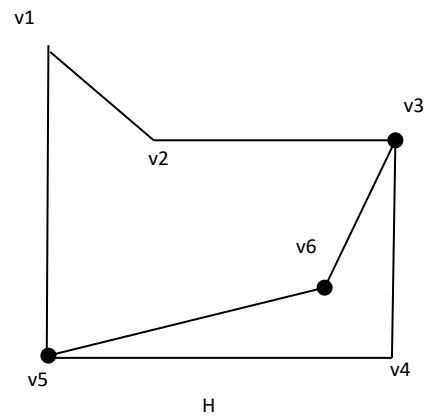
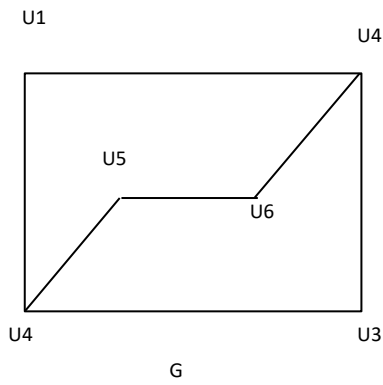
Example 6. All the invariant properties of isomorphic graphs are satisfied. If we Label the vertices a b c d e in G as u_1, u_2, u_3, u_4, u_5 and H we have to label the vertices as follows w, v, u, x, y in H as v_1, v_2, v_3, v_4, v_5 .

To prove the two graphs have the same adjacency $A_G =$

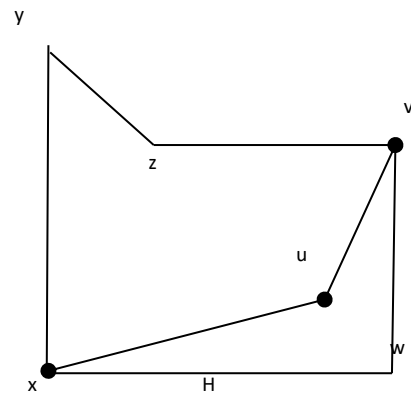
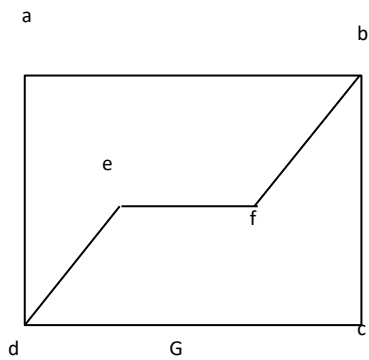
$$\begin{matrix}
 & U_1 & U_2 & U_3 & U_4 & U_5 \\
 U_1 & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} \\
 U_2 & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix} \\
 U_3 & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
 U_4 & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 U_5 & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix}
 \end{matrix}$$

$$A_H^{\square} =$$

$$\begin{matrix}
 & V_1^{\square} & V_2^{\square} & V_3^{\square} & V_4^{\square} & V_5^{\square} \\
 V_1^{\square} & \left[\begin{array}{ccccc}
 0 & 1 & 0 & 1 & 0 \\
 1 & 0 & 1 & 0 & 1 \\
 0 & 1 & 1 & 0 & 1 \\
 1 & 0 & 0 & 0 & 1 \\
 0 & 1 & 0 & 1 & 0
 \end{array} \right] \\
 V_2^{\square} \\
 V_3^{\square} \\
 V_4^{\square} \\
 V_5^{\square}
 \end{matrix}$$



Example 7. Verify the whether the following two graphs are isomorphic or not



$$A_G^{\square} =$$

	a	b_{\square}	c_{\square}	d	e_{\square}	f
a	0	1	0	1	0	0
b_{\square}	1	0	1	0	0	1
c_{\square}	0	1	0	1	0	0
d	1	0	1	0	1	0
e	0	0	0	1	0	1
f	0	1	0	0	1	0

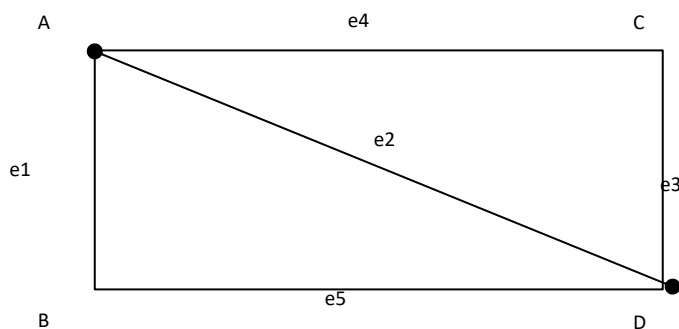
$$A_H^{\square} =$$

	u	v	w_{\square}	x_{\square}	y_{\square}	z_{\square}
u_{\square}	0	1	0	1	0	0
v_{\square}	1	0	1	0	0	1
v_{\square}	1	0	1	0	0	1
x_{\square}	1	0	1	0	1	0
y_{\square}	0	0	0	1	0	1
z	0	1	0	0	1	0

Example 7. Since $A_G^{\square} \equiv A_H^{\square}$ so G and H are isomorphism between G and H established. Incident matrix in undirected graph.

$$A_H^{\square} =$$

	e_1^{\square}	e_2	e_3^{\square}	e_4^{\square}	e_5^{\square}
A_{\square}	1	1	0	1	0
B_{\square}	1	1	0	1	0
C_{\square}	0	0	1	1	0
D_{\square}	0	1	1	0	1



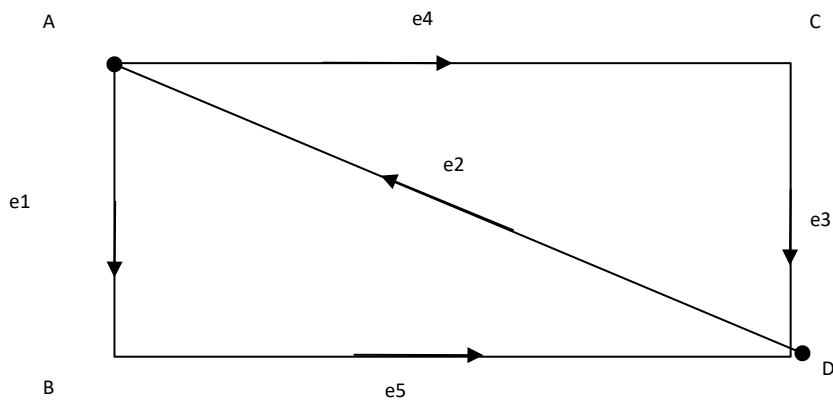
Example 8. Under consideration matrix is incident matrix and called binary matrix or (0, 1) matrix and also called vertex edge incidence matrix and complete matrix

$$A_r^{\square} =$$

$$\begin{matrix} & e_1^{\square} & e_2 & e_3^{\square} & e_4^{\square} & e_5^{\square} \\ A^{\square} & \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix} \\ B^{\square} & \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix} \\ C^{\square} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Incident matrix in case of directed graph, $A_G^{\square} =$

$$\begin{matrix} & e_1^{\square} & e_2 & e_3^{\square} & e_4^{\square} & e_5^{\square} \\ A^{\square} & \begin{bmatrix} +1 & -1 & 0 & +1 & 0 \end{bmatrix} \\ B^{\square} & \begin{bmatrix} -1 & 0 & 0 & 0 & +1 \end{bmatrix} \\ C^{\square} & \begin{bmatrix} 0 & 0 & +1 & -1 & 0 \end{bmatrix} \\ D^{\square} & \begin{bmatrix} 0 & +1 & -1 & 0 & -1 \end{bmatrix} \end{matrix}$$



Example 8. Conversely if we have to obtain a reduced incidence matrix then we have to add 1 or 0 to obtained D row then obtain matrix is called reduced incidence matrix.

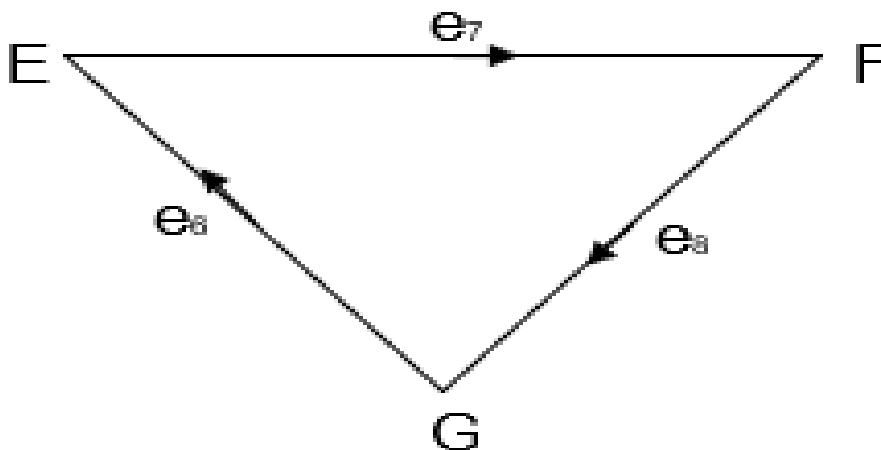
$$A_r^{\square} =$$

$$\begin{matrix} & e_1^{\square} & e_2 & e_3^{\square} & e_4^{\square} & e_5^{\square} \\ A^{\square} & \begin{bmatrix} +1 & -1 & 0 & +1 & 0 \end{bmatrix} \\ B^{\square} & \begin{bmatrix} -1 & 0 & 0 & 0 & +1 \end{bmatrix} \\ C^{\square} & \begin{bmatrix} 0 & 0 & +1 & -1 & 0 \end{bmatrix} \end{matrix}$$

$$A_r^{\square} =$$

$$\begin{matrix} & e_1^{\square} & e_2 & e_3^{\square} & e_4^{\square} & e_5^{\square} \\ A^{\square} & \begin{bmatrix} +1 & -1 & 0 & +1 & 0 \end{bmatrix} \\ B^{\square} & \begin{bmatrix} -1 & 0 & 0 & 0 & +1 \end{bmatrix} \\ C^{\square} & \begin{bmatrix} 0 & 0 & +1 & -1 & 0 \end{bmatrix} \\ D^{\square} & \begin{bmatrix} 0 & +1 & -1 & 0 & -1 \end{bmatrix} \end{matrix}$$

In case of non-connected graph $A_r^{\square} =$



	e_1^{\square}	e_2^{\square}	e_3^{\square}	e_4^{\square}	e_5^{\square}	e_6^{\square}	e_7^{\square}	e_8^{\square}
A^{\square}	+1	-1	0	+1	0	0	0	0
B^{\square}	-1	0	0	0	+1	0	0	0
C^{\square}	0	0	+1	-1	0	0	0	0
D^{\square}	0	+1	-1	0	-1	0	0	0
E^{\square}	0	0	0	0	+1	-1	-1	0
F^{\square}	0	0	0	-1	0	0	0	-1
G^{\square}	0	0	0	0	0	+1	0	-1

3.2. Reduced Incidence Matrix

If we reduce row D from above matrix then obtained matrix called reduced incident matrix. In this case we note the sum of all the column of A_G^{\square} will be zero, Conversely if we have to obtained an incident matrix from a reduced matrix then we have to add (+1, -1) to get summation zero then the matrix will be called reduced incident matrix

4. Conclusions

For any two graphs to be isomorphic if the following four condition must be satisfied. The number of vertices of both the graphs must be the same. The number of edges in both the graphs must be the same. The degree sequence of both graphs must be the same If a cycle of length n is formed by the vertices

$$\{v_1^{\square}, v_2^{\square}, v_3^{\square}, v_4^{\square}, \dots, v_n^{\square}\}.$$

In one graph then a cycle of same length and must be formed by the vertices

$$\{f(v_1^{\square}), f(v_2^{\square}), f(v_3^{\square}), \dots, f(v_n^{\square})\}.$$

In the other graph as well. However, if any condition violates, then it can said that the graphs are surely not isomorphic. Therefore, two graphs, which contain the same number of vertices, connected in the

same way and satisfied all isomorphism conditions said to be isomorphic.

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